Effect of the Imperfect Channel Estimation on Achievable NOMA Rate

Zoltán Belső, and László Pap

***Abstract*—In recent years, Non-orthogonal Multiple Access (NOMA) has been proposed as an alternative to the more traditional Orthogonal Multiple Access (OMA) schemes for mobile communication. In the NOMA method, the resource domains (like power and bandwidth) are not split but shared between the users of the network. The non-orthogonality means that there is cross-talk between the signals of different users, and the interference is either cancelled by a method called successive interference cancellation (SIC) or treated as part of the noise.**

Comparing the achievable capacity region of OMA and NOMA schemes show that NOMA has advantage over OMA. The SIC method requires knowledge of the channel characteristic between the base station and the user. In the ideal case where all the channel conditions are precisely known, NOMA always performs better than or equal to OMA. In real application, the channel characteristic can only be estimated, which can be non-perfect.

In this paper, we will examine the effect of non-perfect channel estimation on the performance of NOMA and will find that in some cases, NOMA still perform better than OMA, but in other cases OMA would perform better.

***Index Terms*—Non-Orthogonal Multiple Access (NOMA), achievable capacity region, non-perfect channel estimation**

I. Introduction

In recent years, there has been increased discussion of NOMA (Non-orthogonal Multiple Access) as a better choice for multi-user communication in comparison to Orthogonal Multiple Access (OMA) schemes [1]–[4]. The basic working principle of NOMA is superposition coding (SC) and successive interference cancellation (SIC). In the case of downlink communication, the base station transmits the superposition of all the signals of all the active users. In the case of uplink communication, all the active users transmit at the same time, and the superposition of these signals is received by the base station.

The receiver, applying SIC, decodes the strongest signal from the superposition first, even when that signal was not meant to be for them. It then re-modulates the decoded signal, applies the known or rather estimated channel condition of that signal, and subtracts it (which is the interference now for the rest of the signals) from the received signals. It then repeats the process until it reaches the signal of interest for them.

It is well-established that in ideal conditions, NOMA performs at least equally, and in most cases better, in some cases much better than a competing OMA schemes, such as Frequency Division Multiplexing Multiply Access (FDMA). By ideal conditions, we mean only additive white Gaussian noise (AWGN) is present in the channel, and the channel condition (both the phase shift and attenuation) is estimated perfectly for all signals.

Many papers discuss the problem of estimating the receiving channel [5]–[7] in both OMA and NOMA cases. In [8], the authors examine the effect of imperfect channel state information (CSI) due to hardware impairments in a cooperative uplink NOMA environment, focusing on sum rate as a metric. The paper [9] discusses the impact of imperfect SIC due to mismatched cancellation order.

In this paper, we discuss the effect of imperfect channel estimation on the effectiveness of SIC, considering that some part of the stronger signal remains as interference after remodulation and subtraction. Our main metric for the two-user scenario is the achievable capacity region. For the many-user scenario, this metric becomes impractical, so we use the sum of the achievable rates as a metric.

There are two cases we have to discuss: One is downlink communication, where a single base station communicates with several users. The users must share both the bandwidth and the power budget available to the base station. The other is uplink communication, where several users try to communicate with a single base station. The bandwidth is also shared in this case, but every user has its own power budget independent of the others.

In the first part of the paper, we are considering a two-user and a base station scenario, and we share the resources between these two users. For each user, we can calculate the capacity of the channel given the bandwidth and power allocated. The capacity of a band-limited channel with a given signal power level (P), bandwidth (W), and noise power spectral density (N_0) [10]:

$$C = W \cdot \log_2 \left(1 + \frac{P}{N_0 \cdot W} \right) = W \cdot \log_2 (1 + SNR) \quad (1)$$

where the noise power in the band is $N = N_0 \cdot W$ and SNR is the signal-to-noise ratio ($SNR = \frac{P}{N} = \frac{P}{N_0 \cdot W}$). Since the resources have to be allocated between the users, allocating more to one of the users (to increase the capacity of its channel) mean there remains less for the other user, hence its channel capacity will decrease. That means there is a region of achievable capacity for the two users.

In all of this cases, we will discuss how this region is changing when the successive interference cancellation (SIC) cannot be performed perfectly due to imperfect channel estimation. This means that some ϵ portion of the cancelled signal remains and considered as part of the noise while decoding the second

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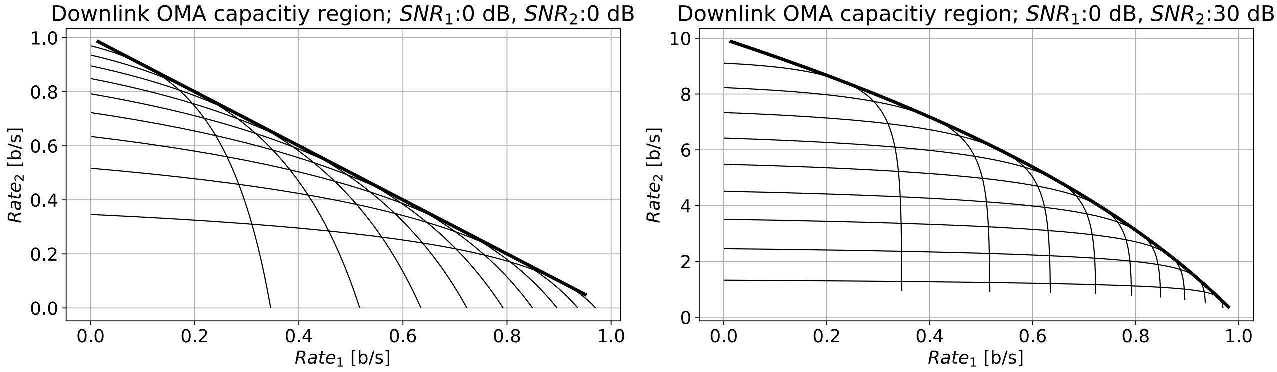

Fig. 1: Downlink OMA (FDMA) capacity region: user1 rate versus user2 rate for different SNR conditions. The thin lines are for fixed values of α ranging from 0.1 to 0.9 (from left to right) [11].

user's signal. As we will see, the effect depends not only on the extent to which the interfering signal remains, but also on what is the ratio of signal powers between the users.

After discussing the two-user case for both uplink and downlink, we extend our analysis by examining the sensitivity of the sum-rate in a multi-user scenario.

This paper is organized as follows: First, we discuss the downlink case, followed by the uplink case, both in a two-user scenario. In each case, we first consider the achievable capacity region in an OMA case (FDMA), then the ideal NOMA case, and finally, the case of imperfect channel estimation, where some interference remains. We then provide a sensitivity analysis for the multi-user scenario. Finally, we conclude with a summary of our findings.

II. DOWNLINK CHANNEL OF TWO USERS

In the downlink scenario, the base station is transmitting two separate signal, one for each user. The baseband signal is denoted by $s_i\ (i = 1, 2)$ with unity power: $E\left[|s_i|^2\right] = 1$. The transmit power for each user's signal is denoted by $p_i\ (i = 1, 2)$. The base station has to split its total transmit power budget between the users, so $p_{tot} = p_1 + p_2$. We can also denote a share coefficient $0 \leq \alpha \leq 1$:

$$\begin{aligned} p_1 =& \alpha \cdot p_{tot} \\ p_2 =& (1-\alpha) \cdot p_{tot} \end{aligned} \tag{2}$$

We denote the total bandwidth of the channel by W.

Each user's channel has a separate channel characteristic $h_i\ (i = 1, 2)$, which is assumed to be a complex number. The absolute value of h_i represents the channel gain, while the phase of h_i represents the phase shift of the channel. These characteristics are independent of each other.

A. OMA case

First, consider the Frequency Division Multiple Access (FDMA) case, where the available bandwidth is divided between the two users. Here, we consider an ideal case where no bandwidth is wasted. We can choose a parameter $\beta,\ (0 \leq \beta \leq 1)$, where one user occupies a $\beta \cdot W$ part of the channel bandwidth, while the other occupies the remaining $(1-\beta) \cdot W$ part, where W denote the total bandwidth of the channel. We consider the division perfectly orthogonal, so that there is no interference between the two users' signal.

The total transmit power of the base station also has to be split between the users.

Here, the maximal rate of communication of every OMA user is [1], [10]:

$$\begin{aligned} R_1 &= \beta \cdot W \cdot \log_2\left(1 + \frac{p_1 \left\|h_1\right\|^2}{\beta \cdot W \cdot N_0}\right) \\ &= \beta \cdot W \cdot \log_2\left(1 + \frac{p_1}{\beta \cdot W \cdot \frac{N_0}{|h_1|^2}}\right) \end{aligned} \tag{3}$$

$$\begin{aligned} R_2 &= (1-\beta) \cdot W \cdot \log_2\left(1 + \frac{p_2 |h_2|^2}{(1-\beta) \cdot W \cdot N_0}\right) \\ &= (1-\beta) \cdot W \cdot \log_2\left(1 + \frac{p_2}{(1-\beta) \cdot W \cdot \frac{N_0}{|h_2|^2}}\right) \end{aligned} \tag{4}$$

See Figure 1 for the case where both user has equal, 0 dB signal to noise ratio (SNR) and for the case where user1 has 0 dB signal-to-noise ratio, while user2 has a much better, 30 dB signal-to-noise ratio. Note that the shape of the convex hull of the capacity region is a straight line for the case of equal channel conditions but a curved line when the two users' conditions are significantly different. The exact shape is derived in [11].

Here, we consider as SNR the full-band noise ($W \cdot N_0$) compared to the total transmit power of the base station as SNR: $SNR_i = \frac{p_{tot}|h_i|^2}{W \cdot N_0}$. That is the SNR for each user when the base station allocates all its power and all the bandwidth to this user, that is when the user is alone.

B. NOMA case

In the power domain NOMA case, both user occupies the whole channel bandwidth, and the base station's transmit power budget is distributed (at some proportion) between them: $p_1 + p_2 = p_{tot}$ or $p_2 = p_{tot} - p_1$, where p_{tot} is the given total transmit power of the base station [10], [12].

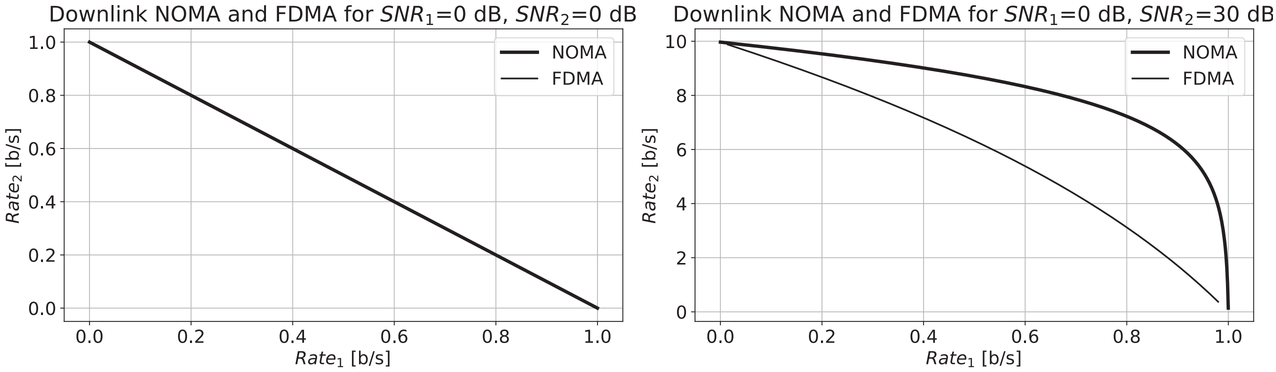

Fig. 2: Downlink NOMA capacity region: user1 rate versus user2 rate for different SNR conditions. For reference, the boundary of the OMA capacity region also plotted.

The transmitted signal by the base station is the sum of the two users' signal:

$$x = \sqrt{p_1} s_1 + \sqrt{p_2} s_2 \tag{5}$$

The received signal by each user ($i = 1, 2$):

$$y_i = h_i \cdot x + w_i \tag{6}$$

where h_i is the complex channel characteristic between the user i and the base station. w_i is the noise sample at the receiver i, assumed to be Gaussian distribution with a mean of 0 and a power spectral density of N_0.

The optimum order of decoding is based on the signal-to-noise ratio (SNR) of the individual signals at each user's receiving end: $|h_i|^2/N_0$. In this decoding order, each user can successively decode any stronger (better SNR) signals and remove them from the received signal (cancellation by re-modulation). The ith user proceeds with successively decoding and cancelling the other user's signal until it reaches its own signal. All the remaining weaker signals are considered as noise or interference.

In the case of only two users, this mean that the first user with better channel conditions receives the other user's signal at a higher power (because it is transmitted at higher proportion of the base station's power budget in order to reach the farther user at a decodable level). Therefore, it decodes the other user's signal first, re-modulates it, and removes it from the original signal. Then, it decodes the remaining signal. During this process, it is assumed that the user can decode the other user's signal without error, but that does not mean that during the cancellation phase it can eliminate it perfectly because during re-modulation, it must consider the effect of the channel on the signal. If the real channel characteristic ($\hat{h}_i = h_i$) were known perfectly, the cancellation could be perfect. If there were some remaining error in the estimated value ($\hat{h}_i \neq h_i$), there would be some interfering signal remaining, proportional to the receiving power of the interfering signal.

1) Perfect channel estimation: In the case of perfect channel estimation ($\hat{h}_i = h_i$), the maximal rate of communication for every NOMA user in a channel with bandwidth W is [1], [10], [12], [13]:

$$R_1 = W \cdot \log_2 \left(1 + \frac{p_1 |h_1|^2}{W \cdot N_0}\right) = W \cdot \log_2 \left(1 + \frac{p_1}{W \cdot \frac{N_0}{|h_1|^2}}\right) \tag{7}$$

$$R_2 = W \cdot \log_2 \left(1 + \frac{p_2 |h_2|^2}{W \cdot N_0 + p_1 |h_2|^2}\right) = W \cdot \log_2 \left(1 + \frac{p_2}{W \cdot \frac{N_0}{|h_2|^2} + p_1}\right) \tag{8}$$

Figure 2 shows the boundary of the achievable capacity region for the NOMA scheme. The first diagram shows the case where both user has equal, 0 dB signal-to-noise ratio (SNR). Note that in that case, we get the same rate limit as in the OMA (FDMA) case. The second diagram shows the case where one of the users has a better, 30 dB SNR. For reference we also plot the boundary for the OMA case. For an exact comparison, when calculating the SNR, we consider the noise power in the total bandwidth ($W \cdot N_0$) compared to the same total transmit power of the base station as in the OMA (FDMA) case, although we get the different rate pairs on the figure by allocating the total base station power divided between the individual users. The difference of in the SNR of the two users represents either the difference in the channel conditions or the local power of the additive Gaussian noise.

2) Imperfect channel estimation: In the case when the channel estimation is not perfect, that is $\hat{h}_i \neq h_i$, after the first user receives and demodulates the stronger signal of the second user, the re-modulation and the cancellation of the received stronger signal cannot be done perfectly. This means that even for the first user, some part of the second user's signal remains as interference. We consider this as if some $\epsilon > 0$ part of the interfering signal power were added to the ever present Gaussian white noise ($W \cdot N_0$):

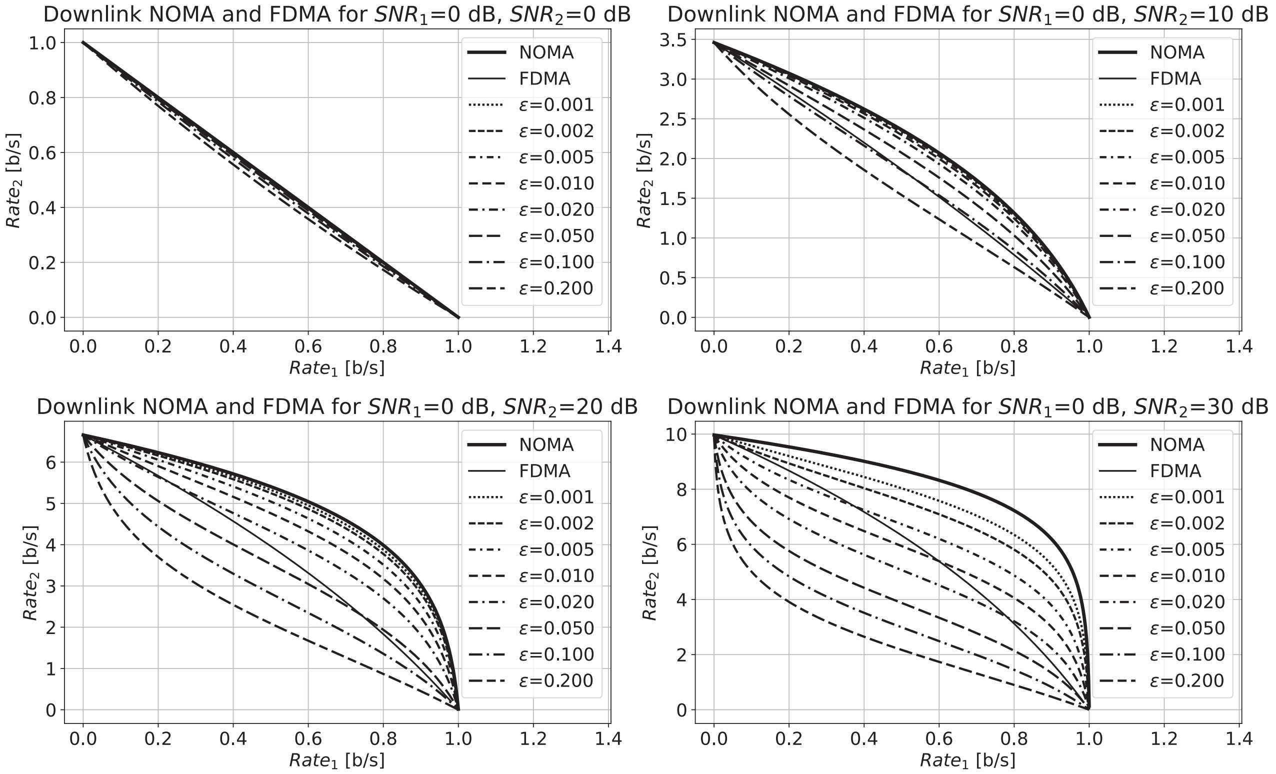

Fig. 3: Downlink NOMA capacity region with imperfect channel estimation: user1 rate versus user2 rate for different SNR conditions. ϵ is the proportion of the remaining interference signal after imperfect cancellation. For reference, the boundary of the OMA capacity region also plotted.

$$R_1 = W \cdot \log_2 \left(1 + \frac{p_1 |h_1|^2}{W \cdot N_0 + \epsilon \cdot p_2 |h_1|^2}\right) \tag{9}$$

$$= W \cdot \log_2 \left(1 + \frac{p_1}{W \cdot \frac{N_0}{|h_1|^2} + \epsilon \cdot p_2}\right)$$

$$R_2 = W \cdot \log_2 \left(1 + \frac{p_2 |h_2|^2}{W \cdot N_0 + p_1 |h_2|^2}\right) \tag{10}$$

$$= W \cdot \log_2 \left(1 + \frac{p_2}{W \cdot \frac{N_0}{|h_2|^2} + p_1}\right)$$

It is easy to predict that the gain of NOMA will decrease as ϵ increases. See Figure 3 for the achievable rates depending on the value of ϵ. In the first diagram, both users have a signal-to-noise ratio (SNR) of 0 dB. In that case there was no gain for NOMA, so for any $\epsilon > 0$, the NOMA rate limit will go below the FDMA rate limit. The second diagram shows the case where one user has a better SNR of 10 dB, while the other has the same poor SNR of 0 dB. In the other two diagrams, one of the users has an even better SNR of 20 dB and 30, respectively. In those cases, NOMA can benefit from the great difference in the power level of the two signals: the interference caused by the weak signal on the decoding of the strong signal is minimal, and the cancellation of the strong signal helps a lot in decoding the weak signal. However, if the cancellation is imperfect, the small portion that is interfering from the strong signal decreases the rate limit of the weak signal because even a small portion of the much stronger signal causes great interference.

III. UPLINK CHANNEL OF TWO USERS

In the uplink scenario, the users transmit independently to the base station. The base station receives the superposition of the users' signals. Let's denote the baseband signal of the two users by s_i $(i = 1, 2)$ with unity power: $E\left[|s_i|^2\right] = 1$. The transmit power of each user is independent of the other and is denoted by p_i $(i = 1, 2)$. We denote the total bandwidth of the channel by W again.

Each user's channel has a separate channel characteristic h_i $(i = 1, 2)$, assumed to be a complex number. The absolute value of h_i represents the channel gain, while the phase of h_i represents the phase shift of the channel. These characteristic are independent of the other user's value.

The received signal at the base station is:

$$y = h_1 \sqrt{p_1} s_1 + h_2 \sqrt{p_2} s_2 + w \tag{11}$$

where w is the noise sample at the receiver, assumed to be a Gaussian distribution of mean 0 and power spectral density of

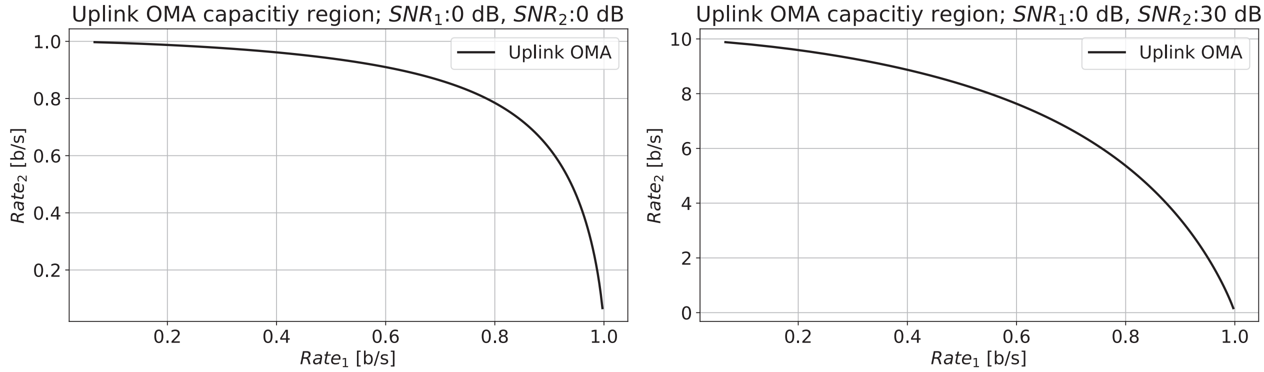

Fig. 4: Uplink OMA (FDMA) capacity region: user1 rate versus user2 rate for different SNR conditions.

N_0 shaped by the receiving filter to the receiving bandwidth. Since there is only one receiver (the base station) in this case, only a single additive noise component applies to the superposition of the two users' signal.

For each user's signal, there are two terms that affect the receiving level: the channel gain and the transmit power. Therefore, without loosing generality, we can assume that the transmitted power is the same for both users ($p_1 = p_2 = p$) and account all the differences in the receive level due to the channel gain.

A. OMA case

For the Orthogonal Multiple Access (OMA) case, let us consider the Frequency Division Multiple Access (FDMA) again: In this scheme, the available bandwidth is divided between the two users. We can choose a parameter $\beta,\ (0 \leq \beta \leq 1)$, where one user occupies a $\beta \cdot W$ part of the channel bandwidth while the other occupies the remaining $(1-\beta)\cdot W$ part, where W denotes the total bandwidth of the channel. We assume the division perfectly orthogonal, so that there is no interference between the two users' signal. The power is not divided between the users; both users are transmitting at full power but only use the allocated part of the bandwidth.

Here, the maximal rate of communication of every OMA user is [1], [10]:

$$R_1 = \beta \cdot W \cdot \log_2\left(1 + \frac{p|h_1|^2}{\beta \cdot W \cdot N_0}\right) \tag{12}$$
$$= \beta \cdot W \cdot \log_2\left(1 + \frac{p}{\beta \cdot W \cdot \frac{N_0}{|h_1|^2}}\right)$$
$$R_2 = (1-\beta) \cdot W \cdot \log_2\left(1 + \frac{p|h_2|^2}{(1-\beta) \cdot W \cdot N_0}\right) \tag{13}$$
$$= (1-\beta) \cdot W \cdot \log_2\left(1 + \frac{p}{(1-\beta) \cdot W \cdot \frac{N_0}{|h_2|^2}}\right)$$

Figure 4 shows the boundary of the capacity region achievable with a classical FDMA case. There are two cases shown: in the first diagram, the users have the same signal-to-noise ratio (SNR): 0 dB, while in the second diagram, one of the users have 30 dB better SNR than the other. Here we consider the in band noise ($\beta \cdot W \cdot N_0$) for calculating the SNR.

B. NOMA case

In the power domain NOMA case, both users occupy the entire channel bandwidth. The base station first decodes the signal of one of the users while considering the interference caused by the other signal as part of the noise. Then, it can remodulate the decoded signal, apply the channel characteristic of the user, and subtract this from the received signal. In an ideal case, it fully eliminates the decoded signal, and the base station can decode the other signal as if it were the only signal. This is called successive interference cancellation (SIC).

In this case, it is not useful to just consider both users transmitting at full power, as it would give us a single point on the capacity plane. Instead, we can trade the channel capacity between the users by scaling the transmit power. Of course, this gives the same result as if we consider the channel conditions as a parameter.

It is usually assumed that the stronger signal is decoded first because eliminating that could help a lot to decode the weaker signal. But that is not the only possibility. Depending on the goal, one may choose to decode and eliminate the weaker signal first. For example, if the goal is to maximize the achievable bit rate of the user with the stronger signal, while letting the weaker user communicating at some lower rate without interfering with the other, that can be achieved by decoding and cancelling the weaker signal first.

As in the downlink case, we can consider two sub-cases: first, when the channel characteristics are perfectly known (perfect channel estimation); and second, when the channel estimation (denoted by $\hat{h_i}$) is imperfect and not equal to the real channel parameter.

1) Perfect channel estimation: If it is the first user's signal that is decoded first, during decoding its signal, the other user's signal is considered as noise [1], [10], [13]:

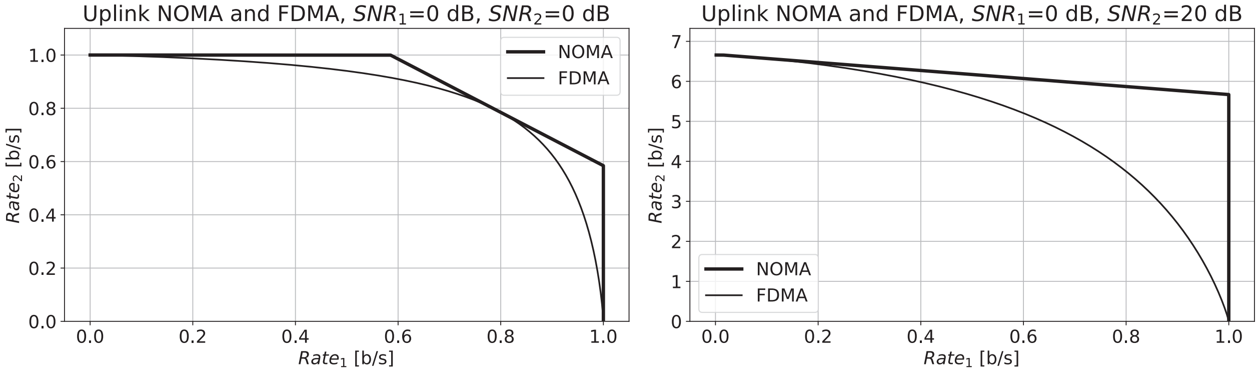

Fig. 5: Uplink NOMA capacity region: user1 rate versus user2 rate for different SNR conditions. For reference, the boundary of the OMA capacity region also plotted.

$$R_1 = W \cdot \log_2 \left(1 + \frac{p_1 |h_1|^2}{W \cdot N_0 + p_2 |h_2|^2}\right) \quad (14)$$

$$R_2 = W \cdot \log_2 \left(1 + \frac{p_2 |h_2|^2}{W \cdot N_0}\right) \quad (15)$$

If it is the second user's signal that is decoded first, the case is the opposite:

$$R_1 = W \cdot \log_2 \left(1 + \frac{p_1 |h_1|^2}{W \cdot N_0}\right) \quad (16)$$

$$R_2 = W \cdot \log_2 \left(1 + \frac{p_2 |h_2|^2}{W \cdot N_0 + p_1 |h_1|^2}\right) \quad (17)$$

In both cases, the sum rate ($R_1 + R_2$) is limited by the sum of the two power (scaled by the channel conditions), which is the total capacity of the channel:

$$R_1 + R_2 = W \cdot \log_2 \left(1 + \frac{p_1 |h_1|^2 + p_2 |h_2|^2}{W \cdot N_0}\right) \quad (18)$$

The achievable capacity region is limited by three factors: for both users, their own maximum power limits the capacity achievable by that user, even in the absence of the other user. That gives us a horizontal and a vertical line in our capacity diagram. At the same time, the sum of the achievable rate of the two users is limited by the sum of their power. That gives us a diagonal line in our capacity diagram. Since all of the conditions must be fulfilled, the capacity region is the convex hull marked by these three fraction lines.

Figure 5 shows the capacity region for two uplink NOMA users. In the first diagram, the two users has equal, 0 dB signal-to-noise ratio (SNR), while on the second diagram one of the users has 20 dB better SNR condition. For reference, we have also plotted the limit of the OMA (FDMA) case from the previous section. Note that in both cases there is one point where the two limits coincides, every other case, the NOMA outperforms the OMA case.

Note, that the two corners of these fraction lines are representing the case where (14), (15) and (16),(17) fulfils with equality, respectively. On the horizontal part of the fraction lines R_2 is constant since the first user is completely eliminated. Similarly on the vertical part R_1 is constant since here the second user is decoded first and completely eliminated, so the capacity of the first user's channel does not depend of the transmitted power of the second user's signal.

2) Imperfect channel estimation: In the case when the channel estimation is not perfect, that is $\hat{h}_i \neq h_i$, after the base station demodulates the signal of the first user, the re-modulation and the cancellation of the received signal cannot be done perfectly. This means that for the second user, there remains some part of the first user's signal as interference. We consider this as if some $\epsilon > 0$ part of the interfering signal power is added to the ever-present Gaussian white noise ($W \cdot N_0$):

$$R_1 = W \cdot \log_2 \left(1 + \frac{p_1 |h_1|^2}{W \cdot N_0 + \epsilon \cdot p_2 |h_2|^2}\right) \quad (19)$$

$$R_2 = W \cdot \log_2 \left(1 + \frac{p_2 |h_2|^2}{W \cdot N_0 + p_1 |h_1|^2}\right) \quad (20)$$

Please refer to Figure 6 for the effect of imperfect channel estimation. In the first diagram, both users have an equal signal-to-noise ratio (SNR) of 0 dB. In this case the situation is symmetrical, and the imperfect cancellation slightly decreases the achievable rate pairs. In the second diagram, one of the users has a better SNR of 10 dB. In this case, imperfect cancellation has a larger effect on the achievable rate pairs. The other two diagrams shows the case where one of the users has an even better SNR of 20 dB and 30 dB, respectively. Please note that these are the cases where NOMA can gain a lot compared to the OMA case.

One thing can be noticed is that the limits for the individual users, which were a horizontal and vertical lines previously, are not straight lines anymore. That is because the transmitted power of the other user, that is decoded first, affects the user whose signal is decoded second due to the imperfect

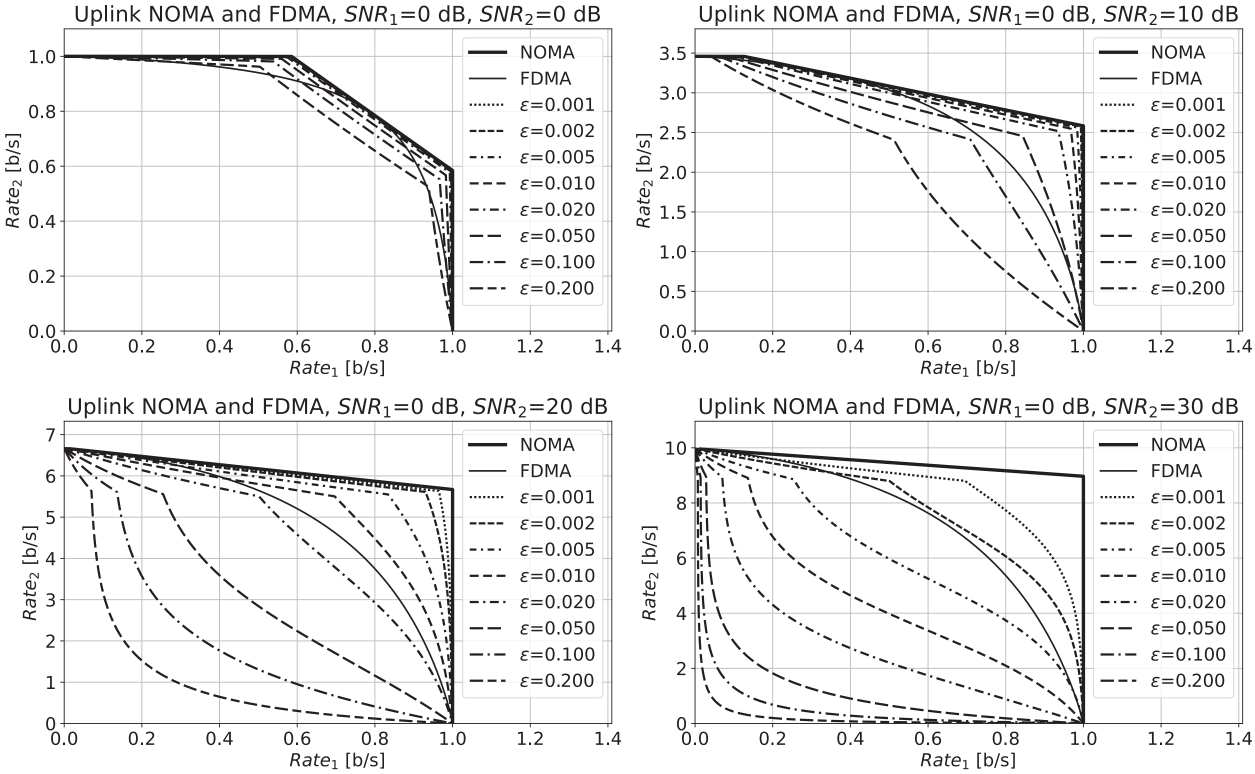

Fig. 6: Uplink NOMA capacity region with imperfect channel estimation: user1 rate versus user2 rate for different SNR conditions. ϵ is the proportion of the remaining interference signal after imperfect cancellation. For reference, the boundary of the OMA capacity region also plotted.

cancellation. We can see that even a small imperfection causes NOMA to partially under-perform the OMA case, and for larger ϵ values, there are hardly any case where NOMA is better. Even in that case, for some regions, NOMA can outperform OMA, but there are large regions where OMA can win.

The diagrams are arranged such that the user with the stronger maximal transmit power is on the vertical axis. One may wonder that at first glance it may seam like the imperfect cancellation affects the stronger user more. To interpret the diagrams correctly, one must keep in mind that they show the boundary of the achievable region, which in most cases corresponds to one of the user not transmitting at maximal power. The imperfect cancellation means that the stronger signal causes strong interference to the weaker signal, so in order to reach a relatively high capacity for the weaker user, the stronger one must decrease power, limiting their own achievable capacity.

IV. Sensitivity

Extending the discussion to more than two users means that the capacity region becomes multidimensional, which makes visualization challenging. A more useful approach is to investigate how sensitive the achievable rate of the users is to small changes in cancellation imperfection.

A. Sensitivity in multi-users case

The inequalities for the multi-user case are (without loss of generality, assuming that the users are numbered in the order of decoding):

$$R_i = W \cdot \log_2 \left(1 + \frac{p_i |h_i|^2}{W \cdot N_0 + \sum_{j=i+1}^{N} p_j |h_j|^2} \right) \tag{21}$$

There is one such equation for each user. For the first user, the signals from all other users act as interference. For the last user, the summation in the denominator is empty since all other users' signals have been canceled.

Considering the imperfection in cancellation, the equations become:

$$R_i = W \cdot \log_2 \left(1 + \frac{p_i |h_i|^2}{W \cdot N_0 + \sum_{k=1}^{i-1} \epsilon_k p_k |h_k|^2 + \sum_{j=i+1}^{N} p_j |h_j|^2} \right) \tag{22}$$

For the first user, the summation term in the denominator that contains ϵ is empty, as there is no signal for the first user to (imperfectly) cancel.

To simplify the following discussion, let's introduce some notation: let $X_i = \frac{R_i}{W}\ln 2$ represent the normalized rate, and $A_i = \frac{p_i|h_i|^2}{W\cdot N_0}$ represent the normalized received power or signal-to-noise ratio (SNR) without interference. Using this notation, the general equation becomes:

$$X_i = \ln\left(1 + \frac{A_i}{1 + \sum_{k=1}^{i-1}\epsilon_k A_k + \sum_{j=i+1}^{N} A_j}\right) \tag{23}$$

To evaluate the sensitivity of the channel to the ϵ_i factors ($i \in [1 \ldots N-1]$), which represent the imperfections in cancellation during SIC processing, we can use the sum rate of the users as a metric:

$$X_{\sum} = \sum_{i=1}^{N} X_i = \sum_{i=1}^{N} \ln\left(1 + \frac{A_i}{1 + \sum_{k=1}^{i-1}\epsilon_k A_k + \sum_{j=i+1}^{N} A_j}\right) \tag{24}$$

We are interested in quantifying the change $dX_{\sum}$ in response to a small change $d\epsilon_i$. This can be expressed through the partial derivatives of $X_{\sum}$:

$$\begin{aligned} dX_{\sum} &= \sum_{l=1}^{N-1}\left(\frac{d}{d\epsilon_l}X_{\sum}\right)d\epsilon_l \\ &= \sum_{l=1}^{N-1}\frac{d}{d\epsilon_l}\left[\sum_{i=1}^{N}\ln\left(1 + \frac{A_i}{1 + \sum_{k=1}^{i-1}\epsilon_k A_k + \sum_{j=i+1}^{N} A_j}\right)\right]d\epsilon_l \end{aligned} \tag{25}$$

$$\begin{aligned} &= \sum_{l=1}^{N-1}\sum_{i=l+1}^{N}\frac{1}{\left(1 + \sum_{k=1}^{i-1}\epsilon_k A_k + \sum_{j=i+1}^{N} A_j\right)^2} \\ &\quad \cdot \frac{-A_i A_l}{1 + \frac{A_i}{1+\sum_{k=1}^{i-1}\epsilon_k A_k + \sum_{j=i+1}^{N} A_j}}d\epsilon_l \end{aligned} \tag{26}$$

Since the sensitivity represents the change in the rate in response to a small imperfection in cancellation, we need to evaluate the derivative at $\epsilon_k = 0$:

$$dX_{\sum} = \sum_{l=1}^{N-1}\sum_{i=l+1}^{N}\frac{1}{1 + \frac{A_i}{1+\sum_{j=i+1}^{N} A_j}}\frac{-A_i A_l}{\left(1 + \sum_{j=i+1}^{N} A_j\right)^2}d\epsilon_l \tag{27}$$

Depending on the relative power levels and whether we are in a high SNR regime ($A_i \gg 1$) or a low SNR regime ($A_i \cong 1$), the imperfection in SIC may change the optimal cancellation order.

B. Sensitivity in two users case

In the case of two users, the sum rate $X_{\sum}$ contains only two terms, and there is only a single ϵ factor:

$$X_{\sum} = \ln\left(1 + \frac{A_1}{1 + A_2}\right) + \ln\left(1 + \frac{A_2}{1 + \epsilon A_1}\right) \tag{28}$$

The sensitivity in this case is:

$$\begin{aligned} \frac{d}{d\epsilon}X_{\sum} &= \frac{d}{d\epsilon}\ln\left(1 + \frac{A_2}{1 + \epsilon A_1}\right) \\ &= \frac{1}{1 + \frac{A_2}{1+\epsilon A_1}}\frac{-A_2 A_1}{(1 + \epsilon A_1)^2} \end{aligned} \tag{29}$$

Since we are considering small imperfections in SIC processing, we take the derivative at $\epsilon = 0$:

$$dX_{\sum} = \frac{-A_1 A_2}{1 + A_2}d\epsilon \tag{30}$$

In the high SNR regime, where $A_2 \gg 1$, so $1 + A_2 \simeq A_2$ it can be further approximated as:

$$dX_{\sum} \cong -A_1 d\epsilon \tag{31}$$

In the low SNR regime, where $A_2 \cong 1$, we can approximate as:

$$dX_{\sum} \cong \frac{-A_1}{2}d\epsilon \tag{32}$$

The sensitivity is practically determined by the power level of the user whose signal we are canceling.

V. CONCLUSION

We have seen that in multi-user communication, we have some degree of freedom in allocating resources (bandwidth and power) to users, which leads to different achievable channel capacities. We can speak of optimal resource allocation in the sense of maximizing the achievable rate of one user while providing some rate for the other. We have discussed the achievable capacity region for both uplink and downlink communication, for both OMA and NOMA schemes. We have seen that a NOMA scheme is attractive and outperforms (or at least equal to) the theoretically optimal OMA case (i.e.: perfect orthogonality, no guard bandwidth needed, no inter signal interference). However, when the channel estimation is imperfect, the successive cancellation of the interfering stronger signal will also be imperfect, leading to a reduced achievable capacity rates for some or both users. In some cases, the degradation due to the imperfection of the channel estimation may result in capacity rates achievable with NOMA being even lower than those achievable with a more traditional OMA scheme.

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